International Journal of Applied Mathematics
& Statistical Sciences (IJAMSS)
ISSN(P): 2319-3972; ISSN(E): 2319-3980
Vol. 3, Issue 2, Mar 2014, 33-36

International Academy of Science, Engineering and Technology
Connecting Researchers; Nurturing Innovations

$\overline{X}(t)$ CONTROL CHART FOR VARIABLE PROCESS DEFECTIVES

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ABSTRACT

In this article a design procedure for $\overline{X}$ and R chart when the process defective is variable in nature is presented. New control limits are derived using the solution of Stochastic differential equation. The procedure is explained by means of examples.

KEYWORDS: VFD, Control Limits, UCL, LCL, CL

INTRODUCTION

When a quality characteristic is measurable on a continous scale the randomness of occurrence of variations in the measurement with respect to time is taken into account and the new control limits are derived using solution of stochastic differential equation. The main objective in any production process is to control and maintain the quality of the manufactured product, so that it conforms to specified quality standards. This is called Process control and is achieved through the technique of control charts pioneered by W.A. Shewhart (1924).

Variable Control Chart Using SDE

$\overline{X}(t)$ and R(t) Charts

No production process is perfect enough to produce all the items exactly alike at any time t. Some amount of variation, in the produced items, is inherent in any production scheme. This variation is the totality of numerous characteristics of the production process viz., raw material, machine setting and handling, operators etc. The variation respect to time is taken into account and the new control limits in the $\overline{X}(t)$ and R(t) charts are derived using solution of stochastic differential equation at any particular time t.

Calculation of $\overline{X}(t)$ and R (t) for Each Subgroup: The mean $\overline{X}_i$ (t) and the range $R_i(t)$ and the standard deviation s_i (t) for the ith sample is calculated. Next the averages of sample means $\overline{\overline{X}}(t)$, sample ranges $\overline{R}(t)$ and sample standard deviations $\overline{s}(t)$ are calculated at any particular time t.

Theorem 1: Let X (t) denote the measurement at time t. $\frac{dX(t)}{X(t)}$, the relative change of measurement evolves according to the Stochastic differential equation

$$\frac{dX(t)}{X(t)} = \mu dt + \sigma dw \text{ where}$$

$$X(t)=X_0 e^{w(t)+(\mu-\frac{\sigma^2}{2})t} \text{ also, } X(t)=X_0 e^{w(t)-\frac{t}{2}}$$

Theorem 2: Control Limits for $\overline{X}(t)$ Chart

The new 3sigma control limits for X(t) are given by

Case 1: When $\mu(t)$ and σ (t) are known. The 3 σ control limits for $\overline{X}(t)$ chart are given by

$$E\left(\overline{X}(t)\right)\pm 3S.E.\left(\overline{X}(t)\right) = \mu(t)\pm\frac{3\sigma(t)}{\sqrt{n}}$$

$$= \mu(t)\pm A\sigma(t)$$

If $\mu'(t)$ and $\sigma'(t)$ are known or specified values of μ (t) and $\sigma(t)$ respectively then

$$UCL_{\bar{x}(t)} = \mu'(t) + A\sigma'(t)$$

$$LCL_{\bar{x}(t)} = \mu'(t) - A\sigma'(t)$$

Case 2: If both μ (t) and σ (t) are unknown then using their estimates $\overline{X}(t)$ and σ (t), we get the 3-σ control limits on the $\overline{X}(t)$ chart as

$$LCL_{\bar{x}(t)} = \overline{X}(t) - A_2\overline{R}(t)$$

$$UCL_{\bar{x}(t)} = \overline{X}(t) + A_2\overline{R}(t)$$

Average Run Length of $\overline{X}(t)$ Chart

The expected number of samples taken or the points plotted on the chart before the shift is detected is the average run length (ARL)

$$ARL=\frac{1}{1-\beta}$$

Where $\beta=P\left[LCL_{\overline{X}(t)} \le \overline{X}(t) \le UCL_{\overline{X}(t)}\right]$

$$\beta=z\left[\frac{UCL_{\overline{X}(t)}-(\mu(t)+k\sigma(t))}{\sigma(t)/\sqrt{n}}\right]-z\left[\frac{LCL_{\overline{X}(t)}-(\mu(t)+k\sigma(t))}{\sigma(t)/\sqrt{n}}\right]$$

$$\beta=z\left[\frac{(\mu(t)+\frac{L\sigma(t)}{\sqrt{n}})-(\mu(t)+k\sigma(t))}{\sigma(t)/\sqrt{n}}\right]-z\left[\frac{(\mu(t)-\frac{L\sigma(t)}{\sqrt{n}})-(\mu(t)+k\sigma(t))}{\sigma(t)/\sqrt{n}}\right]$$

$$\beta = z\left[L - \frac{k}{\sqrt{n}}\right] - z\left[-L - \frac{k}{\sqrt{n}}\right] = P\left[LCL_{\overline{X}} \le \overline{X}(t) \le UCL_{\overline{X}}\right]$$

Hence the average run length of $\overline{X}$ (t) chart coincides with that of $\overline{X}$ chart

Example 1: Construct a control chart for mean using SDE for the following data, comment on whether the production seems to under control, given the starting measurement when t = 0 is 40.(t in hrs.)

Table 1: Variations in 5 Samples

t = 1	**.0001**	**.0002**	**.0003**	**.0004**	**.0001**
t = 2	.002	.001	.003	.004	.002
t = 3	.01	.03	.02	.05	.04
t = 4	.05	.03	.02	.01	.04
t = 5	.06	.07	.06	.05	.07

Table 2: Solution

X(1)	24.263	24.266	24.2685	24.2709	24.2636
X(2)	14.745	14.729	14.759	14.744	14.745
X(3)	9.015	9.197	9.106	9.383	9.289
X(4)	5.6909	5.578	5.522	5.468	5.634
X(5)	3.486	3.5214	3.486	3.4517	3.5214

Control limits are

UCL = 11.457336+(.0507)1.596

LCL= 11.457336-(.0507)1.596

i.e. 11.538, 11.376

The process is out of control as the points corresponding to t=1 and t=2 lie outside the control limits.

CONCLUSIONS

In this paper the variations in the measurements with respect to time is take into account and the new control limits for $\overline{X}(t)$ and R(t) chart are derived using solution of stochastic differential equation. Using the new control limits, a process is said to be in control or out of control at any particular time ‘t’ and it is found that the average run length of $\overline{X}$ (t) chart coincides with that of $\overline{X}$ chart.

REFERENCES

1. DEVAARUL, S (2004), “Certain Studies relating to Mixed Sampling plans and Reliability based Sampling Plans”, Ph.D., Thesis, Department of Statistics, Bharathiar University, Coimbatore, Tamilnadu, India.
2. DEVAARUL, S (2009), “Mixed sampling system with tightened inspection in the second stage”, International Journal of Artificial Integelence, Vol.2, No. S09, pp. 57-65
3. SCHILLING, E.G(1967)General method for determining the OC of variable attributes Sampling plans, Single sided specifications, SD known’, Ph.D. Thesis, Rutgers – The state university, New Brunswick, New Jersey.

4. Lawrance C Evans, An Introduction to Stochastic Differential Equations version 1. Department of Mathematics UC Berkeley.

5. Bernt Oksendal (1945), Stochastic Differntial Equations an introduction with application, Springer, 6^{th} edition